GABARITO COMENTADO

MATEMÁTICA

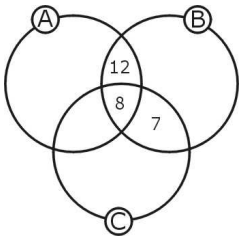
01 Letra B.
n(A) = 8 n(A ∪ B) = 12

$$\underbrace{n(A\cup B)}_{12} = \underbrace{n(A)}_{8} + n(B) - n(A\cap B)$$
$$+ n(B) - n(A\cap B)$$

① - n(B)−n(A ∩ B) = 4

② - n(B − A) = n(B) − n (A ∩ B)
Então: n(B − A) = 4
n[P(B −A] = 2^4 = 16
As partes do vazio já estão no conjunto das partes de (B−A); logo, teremos 16 elementos.

02 Letra A.



n(A ∪ C ∩ B) = 27

03 Letra E.
A região hachurada pertence a B e não pertence a (A ∪ C); logo:
B − (A ∪ B)

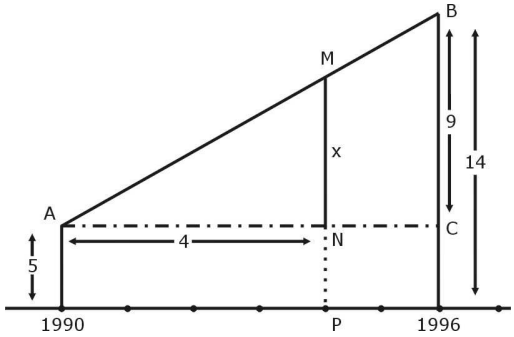
04 Letra B.
A = {1, 3, 5, 7, 9, 11 ...}
B = {−1, 0, 1, 2, 3 ... 9}
C = [5; + ∞ [.
A ∩ B = {1, 3, 5, 7, 9} ⇒ (A ∩ B) −C = {1, 3}
Produto: 1 . 3 = 3

05 Letra B.
$0{,}323232... = \frac{32}{99} \Rightarrow q = 99 \;\; p = 32$
q − p = 67

06 Letra C.
$1999^2 - 1997^2 = \underbrace{(1999 + 1997)}_{3996} . \underbrace{(1999-1997)}_{2}$
= 3996 . 2 .
$1999^2 - 1997^2 - 1998 = 2.3996 - 1998$
= 2 . 2 . 1998 − 1998 = 1998 (4 − 1) =
$\boxed{3 . 1998}$, mas 1998 = 2 . 3^3 . 37
Assim, 3.1998 = 2 . 3^4 . 37 que possui 3 fatores primos distintos.

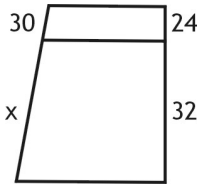
07 Letra E.
$x = \frac{4}{3}$ $y = \frac{1}{6}$ e $x + y = \frac{9}{6} = \frac{3}{2}$
$1{,}333 ... = 1\frac{3}{9}$ $0{,}1666... \; \frac{16-1}{90}$

08 Letra D.



Δ AMN ∞ Δ ABC.
$\frac{x}{9} = \frac{4}{6} \Rightarrow \boxed{x = 6}$. Então, $\overline{MP} = 11$.

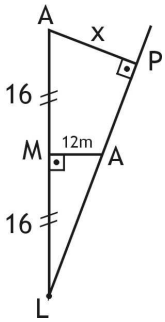
09 Letra B.



$\frac{24}{30} = \frac{32}{X} \Rightarrow \boxed{x = 40m}$

Barreira: 40m − 2m = 38m

10 Letra B.

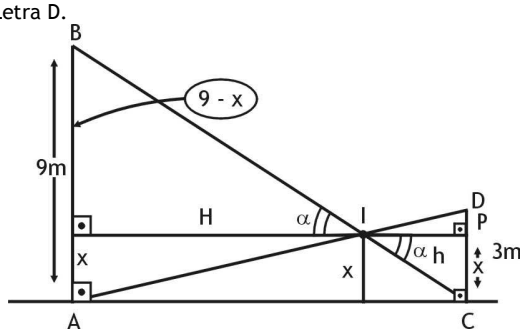


Δ LAP ∞ Δ LMN
$\boxed{\frac{x}{12} = \frac{32}{\overline{LN}}}$

$\overline{LN}^2 = 12^2 + 16^2 \Rightarrow \overline{LN} = 20.$

Então: $\frac{x}{12} = \frac{32}{20}$
x = 19,2m

11 Letra D.

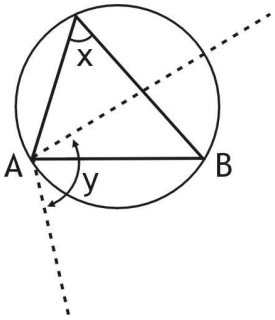


Δ IAB ∞ Δ ICD $\Rightarrow \frac{H}{h} = \frac{9}{3} \Rightarrow \boxed{H = 3h}$

Δ IJB ∞ Δ IPC $\Rightarrow \frac{9-x}{x} = \frac{H}{h} \Rightarrow \frac{9-x}{x} = 3$

$3x = 9 - x \Rightarrow 4x = 9 \Rightarrow x = \boxed{2{,}25m}$

12 Letra E.



Como $x = \frac{\overset{\frown}{AB}}{2}$ e

$y = \frac{\overset{\frown}{AB}}{2}$

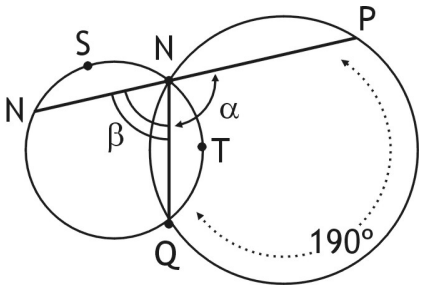
$\boxed{x = y} \Rightarrow x = 66° \; 15'$

13 Letra D.

$\overset{\frown}{ABC} = 18° \Rightarrow \overset{\frown}{AC} = 36°$

$n = \frac{360°}{36°} \Rightarrow \boxed{n = 10}$

14 Letra A.



$\alpha = \frac{190°}{2} \Rightarrow \alpha = 95°$

$\beta = 180° - 95° \Rightarrow$

$\beta = 85° \Rightarrow \overset{\frown}{NQ} = 170°$

$\overset{\frown}{MTQ} = 130°$

$\overset{\frown}{NQ} = 170° \Rightarrow$

$\Rightarrow \overset{\frown}{MSN} = 60°$

15 Letra D.

Se det A = x, det 2A = 4x, e det $A^{-1} = \frac{1}{x}$. Assim,

$16 \cdot \frac{1}{x} = 4x \Rightarrow 4x^2 = 16 \Rightarrow$

$\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$, como $x > 0$

$\boxed{\text{det } A = 2}$

16 Letra E.

det B = 4 $\Rightarrow$ det 2B = 4 . $2^3 \Rightarrow$ det 2B = 32

det (A . 2B) = det A . det 2B = $\underbrace{3 \cdot 32}_{96}$

17 Letra C.

det A = x det (A + A) = det 2A =

= x . 2^2 = 4x

det A^2 = det(A . A) = det A. det A = x^2

Então: $x^2 = 4x \Rightarrow \begin{cases} x = 0 \\ x = 4 \end{cases}$

Como det A $\neq$ 0 $\Rightarrow$ det A = 4

18 Letra D.

$$A^2 = \begin{bmatrix} -1 & 2 & -5 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 & -5 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

$$A^2 = \begin{bmatrix} 3 & -4 & 2 \\ -2 & 3 & -5 \\ 0 & 0 & 1 \end{bmatrix}; XI = \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}$$

$$A^2 - XI = \begin{bmatrix} 3-x & -4 & 2 \\ -2 & 3-x & -5 \\ 0 & 0 & 1-x \end{bmatrix}$$

det $(A^2 - XI) = (3 - x)(3 - x)(1 - x) - 8(1 - x)$

Como det $(A^2 - XI) = 0$, teremos:

$(3 - x)(3 - x)(1 - x) - 8(1 - x) = 0$

$(1 - x)[(3 - x)^2 - 8] = 0$

$1 - x = 0 \Rightarrow x_1 = 1$

$(3 - x)^2 - 8 = 0 \Rightarrow (3 - x)^2 = 8 \Rightarrow 3 - x = \pm 2\sqrt{2}$

$x = 3 \pm 2\sqrt{2} \Rightarrow x_2 = 3 + 2\sqrt{2} \quad x_3 = 3 - 2\sqrt{2}$

Soma: $1 + 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 7$

19 Letra A.

det A = det B = X

det A^t = det A = X

det $B^{-1} = \frac{1}{\det B} = \frac{1}{X}$

$\det\left(\frac{1}{2} \cdot A^t\right) = \left(\frac{1}{2}\right)^n . \det A^t = \left(\frac{1}{2}\right)^n . X$

Então:

$\det\left(\frac{1}{2} \cdot A^t \cdot B^{-1}\right) = \left(\frac{1}{2}\right)^n . X . \frac{1}{X} = \left(\frac{1}{2}\right)^n$

20 Letra E.

Se 97 são homens, 3 são mulheres.

Na nova condição $\begin{cases} \text{Homens} - 96\% \\ \text{Mulheres} - 4\% \end{cases}$; logo,

Como não vão sair mulheres, teremos:

4% _____ 3

96% _____ X $\Rightarrow \boxed{X = 72}$

Devem sair: 97 − 72 = 25 homens.